

## MATLAB Marina: Problem Solving Example 1

### Determining if Thrown Object Hits Target

Write a MATLAB program to determine if a thrown snowball will hit a target.

Problem analysis:

Have we been given enough information to solve the problem? If not, what do we need to be able to solve the problem? Determining if a thrown object hits a target involves projectile motion (physics). The equations of motion in two dimensions are:

$$s_x = s_{x0} + v_{x0} \cdot t + \frac{1}{2} g_x \cdot t^2$$

$$s_y = s_{y0} + v_{y0} \cdot t + \frac{1}{2} g_y \cdot t^2$$

Where  $s$  is position,  $v$  is velocity (speed and direction),  $g$  is the acceleration due to gravity, and  $t$  is time. To determine if a projectile hits a target, we need to know the location of the target relative to the initial location of the projectile, i.e. how far away (horizontal and vertical distance) is the target from the thrower. We will also need to know the initial velocity and initial direction of the projectile (how hard and in what direction was the snowball thrown) and the size of the target.

Assuming that gravity is constant, that the target does not move and is at position  $(s_x, s_y)$ , that the thrower is stationary with initial position  $(s_{x0} = 0, s_{y0} = \text{thrower's height})$ , and that the snowball does not lose any energy due to air resistance:

- The inputs to the program will be the initial velocity  $v_0$  (how hard thrown), initial height (how tall is thrower), and release angle of the snowball (direction thrown).
- The program will determine if the target was hit. To determine if the target was hit, the program must determine if the target is in the snowball's path of motion.
- The program will need to solve for the time it takes for the snowball to reach the target and the height of the snowball at that time, i.e. what is the height of the snowball when it covers the horizontal distance to the target. The program can then check if this height is within the area occupied by the target.

Initial Algorithm:

- Read in the horizontal and vertical distance to the target.
- Read in the initial velocity and release angle of the snowball.
- Determine if snowball hit target.
- Output whether snowball hit or missed target.

Further refining the determination if the snowball hit the target:

- Calculate the  $x$  and  $y$  components of the snowball's velocity.
- Calculate the time it takes the snowball to reach the target.
- Calculate the height of the snowball when it reaches target.
- Determine if snowball height is less than the target height. If yes, the snowball hit the target.

Final Algorithm:

- Read in the horizontal and vertical distance to the target.
- Read in the initial velocity and release angle of the snowball.
- Determine if snowball hit target.
  - a) Calculate the x and y components of the snowball's velocity.
  - b) Calculate the time it takes the snowball to reach the target.
  - c) Calculate the height of the snowball when it reaches target.
  - d) Determine if snowball height is less than the target height. If yes, the snowball hit the target.
- Output whether snowball hit or missed target.

A comment skeleton corresponding to the algorithm is shown in Figure 1a. The coded program is shown in Figure 1b.

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```
% Horizontal and vertical distance to the target

% Initial velocity and release angle of the snowball

% Determine if snowball hit target
% Calculate the x and y components of the snowball's velocity

% Calculate the time it takes the snowball to reach the target

% Calculate the height of the snowball when it reaches target

% Determine if snowball height is less than the target height

% Output whether snowball hit target.
```

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Figure 1a. Comment Skeleton for Snowball Program

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```

% gravity (ft/s^2)
gx = 0.0;
gy = -32.81;

% Initial position of thrower (ft)
sx0 = 0;
sy0 = input("Enter height of thrower in ft: ");

% Horizontal and vertical distance to the target
sx = input("Enter horizontal distance to the target in ft: ");
height = input("Enter height of target in ft: ");

% Initial velocity magnitude and release angle of the snowball
v0 = input("Enter the velocity (magnitude) of snowball in ft/s: ");
theta = input("Enter release angle of snowball in degrees: ");

% Determine if snowball hit target
% x and y components of the snowball's velocity
vx0 = v0*cosd(theta);
vy0 = v0*sind(theta);

% Calculate the time to reach target and
% height of the snowball when it reaches target
ttarget = sx/vx0;
sy = sy0 + vy0*ttarget + 0.5*gy*ttarget^2;

% Determine if snowball height is less than the target height.
% If yes, the snowball hit the target.
if (sy > 0.0 && sy < height)
    message = "Snowball hit target\n";
else
    message = "Snowball missed target\n";
end
% Output whether snowball hit target.
fprintf(message);

```

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Figure 1b. Snowball Program (Omitting Program Comment Header)


To determine if the program operates correctly, one must test the program for a representative set of test cases. Test for a fairly simple case that can be verified in your head first then test for the other cases. For the snowball program, one representative set of test cases is: a hit where the distance is small (< 10 feet), a hit for a distance of around 20 – 50 feet, a miss where snowball comes up short, and a miss where the snowball flies over the target's head.

Some sample results for the snowball program are:

- Height thrower 6 feet, height target 5.5 feet, distance to target 10 feet, snowball velocity 50 fps, and snowball angle 0 degrees. This is a hit, height of snowball at the target is 5.34 feet.
- Height thrower 6 feet, height target 5.5 feet, distance to target 50 feet, snowball velocity 50 fps, and snowball angle 20 degrees. This is a miss, height of snowball at the target is 5.6 feet. The snowball flew just over the target.
- Height thrower 6 feet, height target 5.5 feet, distance to target 50 feet, snowball velocity 50 fps, and snowball angle 10 degrees. This is a miss, height of snowball at the target is -2.1 feet. The snowball fell short.
- Height thrower 6 feet, height target 5.5 feet, distance to target 50 feet, snowball velocity 50 fps, and snowball angle 15 degrees. This is a hit, height of snowball at the target is 1.8 feet.
- Height thrower 6 feet, height target 5.5 feet, distance to target 50 feet, snowball velocity 100 fps, and snowball angle 10 degrees. This is a miss, height of snowball at the target is 10.6 feet. The snowball flew well over the target.

Note: 50 fps is 34.1 mph and 100 fps is 68.2 mph.

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